

## Exercise 28

Plot the gradient vector field of  $f$  together with a contour map of  $f$ . Explain how they are related to each other.

$$f(x, y) = \cos x - 2 \sin y$$

### Solution

Calculate the gradient and call it  $\mathbf{F}$ .

$$\begin{aligned}\mathbf{F} &= \nabla f \\ &= \left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y} \right\rangle f \\ &= \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right\rangle \\ &= \left\langle \frac{\partial}{\partial x}(\cos x - 2 \sin y), \frac{\partial}{\partial y}(\cos x - 2 \sin y) \right\rangle \\ &= \langle (-\sin x), (-2 \cos y) \rangle\end{aligned}$$

The vector field of this gradient is superimposed on a contour plot of  $f(x, y)$ . Notice that the vectors are perpendicular to each of the contours, pointing in the direction of greatest increase.

